Optimality of Distributed Grid Intelligence for Power Distribution

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***Abstract*—The load balancing algorithm used for the Future Renewable Electric Energy Delivery and Management (FREEDM) System power migration is analyzed for optimality. The discussion includes reduction from the bin packing problem, and limitations of this approach. Additionally, the discussion shows a reduction from the fractional knapsack problem, which allows for optimization based upon minimum cost of power, yielding a practical algorithm for distributed load balancing.**

***Index Terms*—Power Migration, Load Balancing, Algorithms, FREEDM**

I. Introduction

As an increasing number of disciplines turn to *smart* technology, classical computer science makes increasingly important contributions. In modern electrical engineering, the so-called *smart-grid technology* requires advanced system techniques and theory. The idea of a "smart power grid" is on the minds of many people from electrical engineers and computer scientists to politicians and security experts [1].

In 2008, the National Science Foundation (NSF) launched an Engineering Research Center (ERC) for a *Green Energy Grid*. Called the NSF ERC for Future Renewable Electric Energy Delivery and Management (FREEDM) Systems, its mission is to develop technology in order to create "an efficient network that integrates alternative energy generation and novel storage combination and scale of energy sources and storage devices" [2]. In FREEDM, neighboring houses and businesses share power with a collective connection to the utility grid. The system is managed distributively; there is no single, centralized, coordinator.

The FREEDM System requires numerous advancements in technology and management systems in order to successfully meet its goal. The focus of this paper is on that management system. Specifically, the details of the system that manages the sharing of power sources between neighboring houses and businesses are analyzed for efficiency. This system is currently experimental and will need additional complexity for further optimization constraints. These issues are discussed and an extension to the current implementation is proposed.

II. Background

A. FREEDM System

The FREEDM System offers a smart-grid solution that will facilitate the seamless integration of renewable energy at the distribution level [3]. This smart-grid will additionally support operation in a disconnected mode; this allows function without a utility grid tie-in. There are several sub-thrust of the project. This paper explains the theoretical operation of the Distributed Grid Intelligence (DGI).

The DGI system operates using distributed algorithms in Intelligent Energy Management (IEM) nodes. IEMs are integrated with SSTs to provide a high-level control system that coordinates changes across the microgrid. IEMs are the core of the DGI development. A Solid State Transformer (SST) is a power electronics device that manages electrical power flow into and out from the FREEDM microgrid, hereafter called the *distribution grid*. Two other terms pertinent to this discussion are the Distributed Energy Storage Device (DESD) and Distributed Renewable Energy Resource (DRER). The DESDs operates as an advanced battery, allowing an IEM to store energy during period of inexpensive power and release energy when power is more costly. The DRERs exist at each site (i.e. an individual's home) within the distribution grid and provide energy generation. Later, these will be referred to as *sources* [4].

B. Single IEM Optimization

For a single IEM, the effective load on the distribution grid at the SST can be expressed as Equation 1 [5].

$$X = X_{Load} - X_{DG} + X_S, \tag{1}$$

where,

- X_{Load} is the SST user load at the SST LVAC terminals,
- X_{DG} is the power output of the distributed energy source, and
- X_S is the battery state (difference in charging and discharging) and is given by:

$$X_S = X_{S,C} - X_{S,D}$$

It follows that the effective load on the utility grid at the grid tie-in can be expressed as the sum of the each individual SST effective loads for every IEM node $i \in I$ as in Equation 2. Let M be the set of IEM nodes with source generation and N be the set of IEM nodes with load demands. Note that these sets are not mutually exclusive.

$$G = \sum_{i \in I} X \tag{2}$$

$$= \sum_{i \in N} X_{Load,i} - \sum_{j \in M} X_{DG,i} + \sum_{i \in I} X_{S,i} \tag{3}$$

$$= \sum_{i \in N} X_{Load,i} - \sum_{j \in M} X_{Gen,j} \tag{4}$$

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Equation 4 is merely a further simplification, which includes the battery state in the load or the generation when it is charging or discharging, respectively. This simplification is used later for readability.

As an initial approach, assume that utility power is always more expensive in terms of monetary cost than operating renewable distributed energy sources. Section III-E considers the energy cost per unit of power, allowing use of utility power during off-peak hours, when it is less expensive. The global optimization then, is to minimize G by migrating available power from the m IEM nodes with available generation sources to the n loads, where $m, n \leq |I|$, such that all power generated within the microgrid is used within the microgrid. This minimizes power drawn from the utility grid, thus minimizing global energy costs.

III. Analysis

A. Energy Migration by Load Balancing

The approach taken by the DGI sub-thrust of FREEDM is to utilize an existing load balancing algorithm originally designed for migrating computational processes among distributed computers [6]. In the DGI implementation, however, instead of migrating processes from one candidate processor to another, electrical load is migrated from one IEM to another. The physical semantics of this are that the receiving IEM node makes adjustments in the local SST to allow more power to flow *into* the distribution grid, while the node that is migrating the electrical load adjusts its local SST to allow more power to flow *from* the distribution grid. The algorithm migrates the loads to the energy, but the physics migrates the energy to the load. These two perspectives are equivalent. The implementation details of the algorithm and simulation system are detailed in [7].

Problem 3.1 (Power Migration Decision Problem): Does a power migration strategy exist that migrates power from m IEM nodes with power supply capabilities to n IEM nodes with electrical load demands resulting in a draw from the utility grid of at most g units.

B. Local and Global Optimization

Sathyanarayana [5] outlines the multi-objective optimization techniques using multi-objective pareto optimization for the operation of the local SST. The optimization constraints are:

1) Minimize peak demand (kW) from the distribution network over the time horizon (e.g., a single day)
2) Minimize the energy demand from the distribution network (kWh) during the user specified '*peak load period*'.
3) Minimize the total cost of energy delivered from the distribution network for the time duration of the study (in dollars).
4) Minimize the energy lost in the DESD converter and the battery implementation of that storage (kWh). The energy loss is calculated over the study time horizon.

Each of these constraints is met, respectively, by the following mechanisms:

1) Utilize as much local generation as possible to meet load demands.
2) Utilize local DESD, when available, to offset demand from distribution grid during the configured '*peak load period*'.
3) Utilize the least expensive energy source available.
4) When utilizing the local DESD, use short cycles or deep cycles as dictated by the DESD to minimize the impact on the lifetime of the battery.

Each of these optimizations apply towards the global optimization. The global optimization is similar to the local optimization, except that the minimization seeks to apply to the utility network rather than the distribution network.

C. Algorithm Analysis

The power load balancing algorithm is based upon the following two lower bounds on the optimal migration, OPT.

1) The average load for which an IEM must provide, $(\sum_{i \in I} X_i)/n$; and
2) The largest electrical load.

Let LB denote the combined lower bound:

$$LB = \max\left\{\frac{1}{n}\sum_{i \in I} X_i, \quad \max_i\{X_i\}\right\}. \tag{5}$$

The implementation of the algorithm uses a greedy approach, migrating power from the node with the greatest available power to the node with the greatest load demand.

Distributed Load Balancing Algorithm

Algorithm 1 DraftSelect - *On Source Node*

1: Order the draft requests arbitrarily.
2: DraftSelect requests in this order, select the next draft on the node that has the greatest amount of load so far.

Algorithm 2 DraftAccept - *On Demand Node*

1: Order the draft select messages arbitrarily.
2: DraftAccept selects in this order, select the next selection message on the node that has the least amount of load so far.

Theorem 3.1: Algorithm 1 achieves an approximation guarantee of 2 for the minimum migration problem.

Proof: Assume the algorithm is operating on a snapshot of the system state with j loads to balance. Let M_i be the node that is the last to receive a load migration produced by the algorithm, where j is the index of the last migration received on this machine. For simplicity, also assume that all IEM nodes have equal generation.

Let $X_{Load,j}$ be the load size at the time when migration j is initiated on M_i. Since the algorithm assigns a load to the least loaded node, it follows that all other nodes have a current load greater than $X_{Load,j}$. This implies that

$$X_{Load,j} \leq \frac{1}{n}\sum_{i \in I} X_i \leq OPT.$$

Further, $\max_{i \in I}(X_i) \leq OPT$. Since all other loads have been scheduled, the worst case is when $X_{Load,j}$ is migrated to the IEM with the maximum load (see Equation 5). Thus, the upper bound on the migration strategy is $X_{Load,j} + \max_{i \in I}(X_i) \leq 2 \cdot OPT$. ■

D. Power Migration Reduction from Bin Packing.

The migration decision problem is closely related to the bin packing problem by the following observation.

Theorem 3.2: There exists a migration strategy with a draw from the utility network of at most g units if and only if n objects of sizes $p_1, p_2, \ldots, p_n$ can be backed into m bins of capacity $\frac{1}{m}(\sum_{i=1}^{n} p_i - g)$ each.

Proof:

$$G = \sum_{i \in I} X = \underbrace{\sum_{i=1}^{n} X_{Load,i}}_{\text{Local Load}} - \underbrace{\sum_{j=1}^{m} X_{Gen,j}}_{\text{Local Capacity}}$$

$$G + \sum_{j=1}^{m} X_{Gen,j} = \sum_{i=1}^{n} X_{Load,i}$$

Then the power supplied is evenly distributed among each of the m IEM nodes with generation:

$$\sum_{j=1}^{m} X_{Gen,j} = \sum_{i=1}^{n} X_{Load,i} - G$$

$$m \cdot x_{Gen,Avg} = \sum_{i=1}^{n} X_{Load,i} - G$$

$$x_{Gen,Avg} = \frac{1}{m}(\sum_{i=1}^{n} p_i - g)$$

where $X_{Load,i} = p_i$ is the net load drawn from the distribution grid and $x_{Gen,Avg}$ is the capacity of each bin. ■

E. Power Migration Reduction from Fractional Knapsack

The power migration problem may also be solved using the approach of the knapsack problem. In the current algorithm implementation, all power sources have a unit cost. However, when considering renewable energy sources, battery replacement costs, emergency generators, and utility grid power cost, it is useful to have varying costs, allowing optimization for *preferred* energy sources. In this context, *preferred* may be based on monetary cost, carbon footprint, or any other basis by which to judge energy sources.

Theorem 3.3 (Power Migration to Knapsack): There exists a migration strategy with a draw from the utility grid of at most g units if and only if m objects of value $v_1, v_2, \ldots, v_m$ and weights $w_1, w_2, \ldots, w_m$ can be placed into a knapsack such that $\sum_{i=1}^{m} w_i + g = L$, where L is the capacity of the knapsack and equal to the sum of the loads: $L = \sum_{j=1}^{n} X_{Load,j}$.

Proof: Theorem 3.3 follows from a simple renaming of variables. The m IEM nodes with a power generation are represented by $w_1, \ldots, w_m$ and the global load demand is L. Then, g is the remaining draw on the utility grid. In the base case above when any distribution grid source costs less than the utility grid source, set $v_1 = v_2 = \ldots = v_m \leq v_{util}$, where v_i represents the cost of each of the m power sources and v_{util} is the cost of drawing power from the utility grid. In any case, the objective is to minimize the value of the knapsack. ■

F. Power Migration Reduction from Fractional Knapsack

Any greedy approximation algorithm is bounded as before on the interval $(OPT, 2 \cdot OPT]$. However, power migration does not need to be performed in whole units. In fact, to improve stability and efficiency, the DGI implementation uses incremental steps, currently defined at 1 kW [7]. This approach lends well to a modification of the knapsack problem known as the *fractional knapsack problem*.

This modification allows items to be broken into fractional units, in order to maximize value. The greedy approach, then, is to place items of the highest value (lowest energy cost) into the knapsack until the best choice is exhausted or can no longer fit. Once the item can no longer fit, break it into pieces to fill the remaining space. Once the best item is exhausted, begin filling the next best item. Clearly, as the size of the pieces approaches zero, the algorithm result approaches OPT. Finding the optimal solution falls into a class of problems known as NP-Hard [8]. These sort of problems have no known efficient means of being solved. Because of this, the size of the pieces is limited to an approximation interval.

Let C be the cost of the least expensive power source (e.g. $C = \min_{x \in X_{Gen}}(cost(x))$). Then mC is a lower bound on the cost that can be achieved from any solution. Let ϵ be a parameter of the amount of error in the solution. A polynomial-time approximation algorithm can then be devised in terms of m and $1/\epsilon$.

Least Cost Fractional Knapsack

Algorithm 3 Least Cost Fractional Knapsack

1: Given $\epsilon > 0$, let $K = \frac{\epsilon C}{m}$.
2: For each source s_i, define $cost'(s_i) = \left\lfloor \frac{cost(s_i)}{K} \right\rfloor$
3: Add up to K entries of each source in increasing order of $cost'(s_i)$ into the set S', such that $\sum_{s \in S'} cost'(s) \leq L$.
4: Output S', the least cost set.

Theorem 3.4: The least cost set, S' output by the algorithm satisfies: $cost(S') \leq (1 + \epsilon) \cdot OPT$

Proof: Let O denote the optimal set. For any source s, because of rounding down, $K \cdot cost'(s)$ can be larger than $cost(s)$, but not by more than K. Therefore,

$$K \cdot cost'(O) - cost(O) \leq mK$$

Line 3 of the algorithm must return a set at least as good as O under the new costs. Therefore,

$$\begin{aligned} cost(S') &\leq K \cdot cost'(O) \leq cost(O) + mK = OPT + \epsilon C \\ &\leq (1 + \epsilon) \cdot OPT, \end{aligned}$$

where the last line follows the observation that $OPT \geq C$ [9]. ∎

As theorem 3.4 shows, there is a tradeoff in runtime for a higher degree optimality. Specifically, the algorithm runtime is bounded by O($\frac{|M|}{\epsilon}$). As $\epsilon \to 0$, the runtime approaches infinity. For an NP-Hard problem, this is to be expected. By utilizing this fractional knapsack approach in the drafting algorithm, it will guarantee optimal results within ϵ of OPT. These results are important to show that the FREEDM system is operating efficiently.

IV. Conclusion

This paper presented a reduction of the DGI power management system from classic problems in Computer Science for the FREEDM System. This result places bounds on the optimality of the DGI system in terms of optimization work on the bin packing and the knapsack problems. Specifically, the fractional knapsack reduction shows that the FREEDM system can guarantee efficiency on optimization constraints.

The results in this paper provide a theoretical result that ties future development of the DGI system with optimizations on any given objective function or functions. This flexibility will allow optimization on future economical, ecological, or even humanitarian goals as defined by the final implementation.

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