The Maximum Flow Algorithm Applied to the Placement and Distributed Steady-State Control of FACTS Devices

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Abstract

The bulk power system is the largest man made network and its sheer size makes controlling it an extremely difficult task. In this paper, a method to control the network using FACTS devices set to levels determined by a maximum flow (max-flow) algorithm from graph theory is applied to the power system for FACTS device placement and scheduling. A distributed max-flow is introduced to coordinate the actions of the FACTS devices. Finally, the appropriateness of the maximum flow algorithm for power flow control is discussed.

1 Introduction

The bulk power system grid is arguably the largest man-made interconnected network in existence. The sheer size of the power network makes control an extremely difficult task. Cascading failures are the most severe form of contingency that can occur in a power system. A typical cascading failure involves transmission line overloads. In a system where there is a large directional power flow from one region to another, the loss of a single transmission line can instigate a cascading failure. If the first transmission line is lost, the power it carried must be shunted across remaining transmission lines. The redirected power may cause one or more of these transmission lines to overload. If not immediately mitigated, a contingency may lead to a second, a third, and so on, until load must be shed to stabilize the system.

The family of “flexible AC transmission system” (FACTS) devices holds considerable promise as network-embedded controllers. Distributed FACTS controllers in the network can help alleviate the overload problem by directing extraneous power flow away from highly loaded lines. Although one of the most promising applications of FACTS devices is to use them to regulate active and reactive power flow across critical transmission corridors, transmission providers have been reluctant to install them due to their cost and lack of systematic control paradigms. Since grid control has historically been decentralized because of geographic and regulatory constraints, and FACTS devices are naturally decentralized due to their wide geographic distribution throughout the transmission system, they are good candidates for distributed control methodologies. In addition, the rapid expansion of communication technologies, including optical and wireless communication, is making it possible to incorporate real-time, distributed, decision-making processes over a widespread area using a variety of information. As such, distributed FACTS control can be designed to work cooperatively to maintain long-term power system security.

In addition to the development of distributed control procedures, the placement of the FACTS devices within the transmission system is another key issue. For the FACTS devices to achieve maximum control and resiliency to all possible contingencies, the FACTS devices must be placed along critical pathways. The placement of the devices must consider the network topology and resultant power flows after any possible system contingency. Placement alone is only part of the solution. Before FACTS devices can be implemented on a wide-scale basis, they must also coordinate their actions with each other dynamically and rapidly coordinate in the event of a contingency; therefore, a static setting is not sufficient.

One promising technique that addresses all of these issues is the graph theory-based max-flow technique, originally used for transportation control and allocation. This approach was first proposed for power systems by the authors in [1]. The max-flow algorithm finds a set of flows through a network that approximate the power system flows. If this estimate is good enough, it is then used as a basis to set the controls of the FACTS devices. The power system then responds to these settings to stabilize the system. This paper introduces a graph-theory-based maximum flow distributed algorithm to identify critical transmission corridors and control the FACTS devices to adjust the power flow to avoid cascading failures. The primary contributions of this paper are:

1. the use of the max-flow algorithm to adaptively control the FACTS devices in the event of contingencies to maintain system security,
2. the use of the max-flow algorithm to determine the placement of FACTS devices in the power system,
3. the introduction of a distributed max-flow algorithm to coordinate the actions of the FACTS devices in a decentralized network, and
4. the appropriateness of applying max-flow to a power system.

2 Maximum Flow and the Power System Network

This section will give an introduction to the maximum flow (max-flow) algorithm [2]. The algorithm will both be applied to power system power flow control and also as a basis in which to place and coordinate the FACTS devices.

In the max-flow algorithm, the power flow in a network is modeled as a directed graph flow problem with a directed graph $G(N, A)$ modeling the power network. The set of nodes, N, corresponds to the buses of the power network. The power flow between buses $n_i, n_j \in N$ is represented by an arc $a_{ij} \in A$. Each arc is assigned a weight, u_{ij}, denoting the maximum allowable power flow through that particular line. Initially, the weights are set as the steady state power flow values of the system so that the max-flow approximates the actual power flows. The formal max-flow algorithm is given in Figure 1. The algorithm works by successively assigning power flow $f(a_{ij})$ to arcs along a directed path from s to t until no more power can be added.

2.1 Modeling Power Flow Using Max-Flow

An example of a power system modeled as a directed graph $G(N, A)$ is shown in Figure 2. The power system's steady state active power flows are shown for each transmission line. This power system can be equivalently represented as a directed graph as shown in Figure 3. For the basic max-flow algorithm there are two special nodes, the virtual source (s) and the virtual sink (t), representing the combination of the generator(s) and load(s), respectively. The each line out of the virtual source has a capacity that matches the generation of the connected node. Each line into the virtual sink represents the load demanded by the connected node. The graph in Figure 3(a) shows that at the initiation of the max flow algorithm, each arc initially has 0 units of assigned power.

Constrained by the power flow equations, each node in the graph must also satisfy $\Sigma P_{in} = \Sigma P_{out}$, except for the virtual source and sink nodes. The virtual sink may be considered the network "ground" node. The nodes s and t, together with G, form the graph $G'(N', A')$. The graphs in Figure 3 are intended to illustrate a very simple interpretation of the max-flow algorithm that maps the active power flow from the generators (indicated by a single source) to the loads. The appropriateness of this approach is addressed in Section 3.

This model is useful for the case when a line is outaged due to a contingency, and the resulting power flow stresses the network. If too much load is drawn over lines with inadequate capacity then the lines may successively overload and trip off-line, causing a cascading failure. In this scenario, a method is needed to rapidly re-balance the power by either re-directing the power flow across transmission corridors with greater capacity, or by shedding load to maintain adequate power to the remaining loads. By modeling this problem as the maximum flow in a directed graph the FACTS device power flow settings can be determined. Figure 4 shows the system of Figure 3 with flow settings to compensate for the loss of a line; without increasing capacity of the remaining lines, the max-flow tells us some loads cannot be satisfied.

The power system does not actually (nor is it expected) to behave precisely as the max-flow algorithm predicts. Line losses, reactive power flow, and system nonlinearities cause the power to flow differently; however the max-flow scheduling can be used to determine the setting of the FACTS devices. The premise of this approach is that if the FACTS devices are appropriately placed throughout the system, then acting in a coordinated fashion via the max-flow algorithm, they can control the power flow through critical transmission corridors. The FACTS devices will subsequently be able to have significant effect on the power flow across the non-FACTS corridors such that the impact of contingencies on overloads can be substantially mitigated.

3 FACTS Placement and Control Settings

The max-flow algorithm also provides an excellent mechanism for determining the placement of FACTS devices in the power system. The placement of series devices is considerably different than the placement of shunt devices, since series devices impact active power, whereas shunt devices tend to impact reactive power more significantly. Fairly comprehensive studies have been performed to determine the optimal placement of StatComs due to their similarity to SVCs and static capacitor banks [4] [5], but these are not appropriate for the placement of series devices (SSSC and UPFC) in the bulk power system. A variety of approaches have been proposed for placing series devices (usually considering the TCSC) in the system, but little comprehensive work exists that determines the device controls with respect to changes in system topology or loading.

Many utilities considering investment in FACTS devices would like assurances that the devices can be placed and controlled to mitigate any foreseeable contingency. To meet this objective, a useable FACTS placement algorithm must:

- minimize the number of required FACTS devices,
- minimize the number of overloaded lines, and
- provide the FACTS settings (controls) to minimize an economic objective such as fuel costs, line losses, etc.

and this minimization must occur **across the set of all possible system contingencies**. In other words, the placement of a minimum number of FACTS devices **and** their respective settings must be chosen such that for any given contingency, no system lines are overloaded and any economic constraints are met. Several authors have achieved good results on portions of this problem, but to date, no one has attempted to perform this optimization over the entire set of possibilities. This is an important distinction. Most of the placement algorithms for series devices typically consider only *one* set of system flows and/or topology. In a real system, however, it is necessary to consider the placement of the devices such that they will provide power flow control regardless of system topology (i.e. loss of line, load, or generator). To accomplish this, the setting of the devices must be determined *at the same time* as the placement of the devices is determined. This is a large and very complex optimization problem that cannot easily be solved with traditional tools. The max-flow algorithm provides a means to determine both placement and setting simultaneously.

Previous work in determining the optimal location of FACTS devices can be primarily categorized into the following methods:

- Sensitivity-based [6]-[10], or
- Stochastic (Genetic algorithm-based) [11]-[17].

Most of the FACTS placement algorithms have been applied to a system only at steady-state, therefore the control settings of the devices are chosen for a particular topology, load, and generation profile and may also be computationally time-consuming. With the exception of [10], these methods do not consider the placement and setting of the devices over the entire set of possible contingencies, ensuring that no lines are overloaded for any loss of line. But this is exactly the FACTS application that is of interest to transmission service providers. The max-flow algorithm provides a cohesive approach for placing **and** controlling the devices to accommodate any possible contingency.

The implementation of the max-flow algorithm on the power grid is accomplished by using the FACTS devices to enforce the desired flow across each line. The FACTS devices are assumed to be a series device, such as a UPFC, SSSC, or TCSC, that can control the active power flow across the line. The objective is to find the optimum placement of the minimum number of FACTS devices to realize the max-flow algorithm. To properly control the network, the devices need to be arranged in such a way that the grid remains stable under all possible single line contingencies. Simplistically assume that a FACTS device can be placed on every line in the system where the max-flow algorithm would provide the control setting for each device regardless of contingency to ensure no line overloads. This is not a realistic assumption since FACTS devices are too expensive to place on every line and max-flow does not account for line losses. The max-flow algorithm, however, can be used to identify critical corridors across contingencies on which the FACTS devices will provide the greatest impact on system flow.

In this approach, each contingency is analyzed separately using a "greedy" strategy. As each contingency is applied, a FACTS device is placed on the line that is the most overloaded as a result of the outage. The amount of overload can be chosen as absolute value of overload (in MW), as a percentage of the base value, or as a percentage of the maximum capacity of the line. The criteria used will result in slightly different enumeration of the FACTS devices, but not surprisingly, those lines which are chronically overloaded will surface regardless of the criteria used. A FACTS device is then placed on the line with the greatest overload and its capacity is set to the value determined by the max flow algorithm. In the power flow, the FACTS device is modeled as in [18]. This process is repeated for the given contingency until the maximum number of desired FACTS devices have been placed. Then the algorithm is repeated for the next contingency. The max-flow algorithm is applied to the 118 bus, 179 line test system shown in Figure 5.

This placement strategy results in the placement of FACTS devices as shown in Figure 6. The horizontal axis indicates the line that has been outaged. The dots indicate the line where a FACTS device should be place for that particular outage based on percent overload. Note that the figure has a diagonal tendency. This indicates that the overloaded lines tend to be near the outaged line, which is the intuitive result. However, there are also lines that are chronically overloaded, indicated by the FACTS placement on certain lines denoted by the frequency of dots in the vertical direction. A summary of the chronically overloaded lines is given in Table 1. This table lists the line number (corresponding to Figure 6) and the corresponding sending and receiving end bus numbers (corresponding to Figure 5) of the fourteen most chronically overloaded lines. Note that many of the chronically overloaded lines are in close proximity to one another, frequently caused by the same re-directed power flow as a result of the line outage. By using FACTS and the maximum flow algorithm, it is often possible to mitigate adjoining line overloads simultaneously.

The maximum flow algorithm indicates the amount of active power flow that should be carried by all lines in the system; however, this is impossible to physically enforce, unless each line in the system has a FACTS device on it. FACTS devices do enforce the power flow on the critical lines identified. It is theorized that once the critical lines are set, the remaining active power flow in the system will tend to flow according to the maximum flow pattern.

To test this theory, the proposed method is applied to the 118 bus system for a varying number of FACTS devices using load flow to determine the power flow on uncontrolled lines. The results are summarized in Table 2. This table indicates that in the base case (with no FACTS devices) there are 123 (out of a possible 179 total lines) line outages that produce at least one line overload. As FACTS devices are added, the number of cases of overloads decreases significantly. For a placement of four FACTS devices, there are only 19 cases of overload. This means, that out of a possible 179 line outages, only 19 of these outages now cause a line overload. The other 160 line outages (roughly 90%) do not produce a significant change from the base load flow case. Note that the application of just a single FACTS device reduces the number of overload cases from 123 (roughly 70% of all outages) to only 34 cases (20%). These results are achieved through proper placement of the FACTS device and the application of the maximum flow algorithm to determine the active power flow control of each FACTS device. Note, however, that there is also a point at which the benefit of applying more FACTS devices saturates – in this test system, that point is about five devices. The decrease in line overloads saturates after this point, where the decrease in overload cases is roughly one case per FACTS device.

4 A Distributed Version of the Max-Flow Algorithm

In practice, the max-flow control algorithm cannot be run from a single centralized site, but must be run at each FACTS site. The devices are inherently distributed throughout the system; therefore, the original max-flow algorithm must be modified. A modified max-flow algorithm appropriate for decentralized control is presented in this section.

To provide real-time power flow scheduling, the proposed graph theory-based max-flow strategy is implemented such that it can respond to contingencies in real-time over a wide geographic area. The only way to accomplish this is to devise a distributed algorithm in which the FACTS devices cooperatively reach an agreement on their individual settings. Using the communication and computational processes of the FACTS devices along with a network interconnection, the max-flow is implemented as a distributed long-term control algorithm.

Much of the previous work in FACTS power flow control has concentrated on determining *a priori* how power is distributed in the network from each generator. The proposed distributed max-flow algorithm computes the actual flow balance needed to prevent cascading failures in a distributed real-time manner. With distributed max-flow, the algorithm explores augmenting flow paths, simultaneously, from each generator by passing probe and response messages over the communications links. Winning paths are selected for the resulting power flow on a "first discovered" basis. Each FACTS device runs the distributed maximum flow algorithm shown in Figure 7 which is a modification of the one from [20], which is based upon the idea from [2]. The major change from [20] is that flow paths are selected from all generators concurrently.

To use the new max-flow algorithm for the control algorithm, the power problem must be described in appropriate terms. The arc capacities are initially set to the steady-state power flow values. By using the steady-state values, the maximum flow algorithm cannot deviate greatly from what would happen naturally. If max-flow cannot satisfy the loads with these capacities, then changes are made to try to satisfy the load. First, all of the arc capacities are increased to the actual corresponding power flow capacities. The maximum flow algorithm is then restarted with the unsatisfied sink nodes as source nodes and source nodes as sink nodes. The maximum flow algorithm changes slightly to compensate for the change in source and sink nodes. When the arc flows are modified, the negative value of the Δ is added to or subtracted from the arc flow. The final change to the algorithm is in the calculation of the Δ values to be $\Delta_{ij} \leftarrow u_{ji} - f_{ji}$. The new max-flow algorithm finds solutions that satisfy all loads, if possible, using the FACTs devices embedded in the power network.

The FACTS devices remain coordinated through passing messages over a communication network. To meet the real-time constraints the number of messages needs to be minimized. Tests were conducted with the max-flow algorithm described. The program was executed on a cluster consisting of 16 Sun Ultra 10's and 16 Sun Ultra 5's each with 100 Mega-bit Ethernet connection. The workstations are connected through a Catalyst 3200 switching network with a 2.1 Gb/s backplane. Using 5 workstations to represent the FACTS devices and another 19 to represent generators, the program requires 20 seconds to execute. Other max-flow algorithms, such as [21] can be applied to improve this speed for distributed FACTS control.

5 Conclusions

This paper presents a novel method for controlling the active power flow through the power system using FACTS devices. This method is based upon the graph-theoretic maximum flow algorithm to determine the scheduling of active power flow on each line in the system. The maximum flow algorithm can actively compensate for line outages by redirecting power flow to avoid cascading failures. The maximum flow algorithm is implemented by using FACTS devices to enforce the active power flow on critical corridors in the power system. The 118 bus system was used as a test bed to show the effectiveness of the proposed placement and scheduling algorithms. The use of the combined FACTS and maximum flow algorithm was able to significantly reduce the number of line overloads as the result of line outages.

Work in this area continues in developing a fault-tolerant distributed maximum flow algorithm that can be implemented in practice.

6 Acknowledgments

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References

[1] A. Armbruster, B. McMillin, and M. L. Crow, “Controlling power flow using FACTS devices and the Max Flow algorithm,” *Proceedings of the International Conference on Power Systems and Control*, Abuja, Nigeria, December 2002.

[2] L. Ford and D. Fulkerson. Max Flow Through a Network. *Can. J. Mathematics*, pages 399–404, 1956.

[3] D. Fulkerson and G. Dantzig. Calculations of Maximum Flow in Networks. *Naval Logic Quart.*, 2:277–283, 1955.

[4] R. Messina and M. Nayebzadeh, “An efficient placement algorithm of multiple controllers for damping power system oscillations, ” *Proceedings of the 1999 Power Engineering Society Summer Meeting*, Seattle, WA, July 1999.

[5] N. Martins and L. Lima, “Determination of suitable locations for power system stabilizers and static VAR compensators for damping electromechanical oscillations in large scale power systems,” *IEEE Transactions on Power Systems*, vol. 13, no. 2, May 1998.

[6] R. Rajaraman, et. al, “Determination of location and amount of series compensation to increase power transfer capability,” *IEEE Transactions on Power Systems*, vol. 13, no. 2, May 1998.

[7] K. S. Verma, et. al, “FACTS device location for enhancement of total transfer capability,” *Proceedings of the 2001 IEEE Power Engineering Society Winter Meeting*, Columbus, OH, January 2001.

[8] S. N. Singh, et. al, “A New approach for placement of FACTS devices in open power markets,” *IEEE Power Engineering Review*, September 2001.

[9] T. Orfanogianni, et. al, “Steady-state enhancement via optimal utilization of thyristor-controlled series capacitors, *IEEE Transactions on Power Systems*, (to appear).

[10] Y. Lu and A. Abur, “Static security enhancement via optimal utilization of thyristor-controlled series capacitors,” *IEEE Transactions on Power Systems*, vol. 17, no. 2, May 2002.

[11] P. Paterni, S. Vitet, M. Vena, and A. Yokoyama, “Optimal location of phase shifters in the French network by genetic algorithm,” *IEEE Transactions on Power Systems*, vol. 14, no, 2, February 1999.

[12] T. S. Chung, Y. Z. Li, “A Hybrid GA approach for OPF with consideration of FACTS devices,” *IEEE Power Engineering Review*, February 2001.

[13] S. Gerbex, R. Cherkaoui, A. Germond, “Optimal location of multi-type FACTS devices in a power system by means of genetic algorithms,” *IEEE Transactions on Power Systems*, vol. 16, no. 3, August 2001.

[14] L. Lai, J. Ma, “Power flow control with UPFC using genetic algorithms,” *International conference on Intelligent System Applications to Power Systems*, 1996.

[15] H. Leung and T. Chung, “Optimal power flow with a versatile FACTS controller by genetic algorithm,” *Proceedings of the 5th International Conference on Advances in Power System Control, Operation, and Management*, Hong Kong, October 2000.

[16] N. Li, Y. Xu, H. Chen, “FACTS-based power flow control in interconnected power systems,” *IEEE Transactions on Power Systems*, vol. 15, no. 1, February 2000.

[17] W. Feng, G. Shrestha, “Allocation of TCSC devices to optimize total transmission capacity in a competitive power market,” *Proceedings of the 2001 IEEE Power Engineering Society Winter Meeting*, Columbus, OH, January 2001.

[18] D. J. Gotham, G. T. Heydt, “Power flow control and power flow studies for systems with FACTS devices,” *IEEE Transactions on Power Systems*, vol. 13, no. 1, pp. 60-65, February 1998.

[19] R. Ahuja and J. Orlin. A Fast and Simple Algorithm for the Maximum Flow Problem. *Operations Research*, 37(5):748–759, 1989.

[20] T.-Y. Cheung. Graph traversal techniques and the maximum flow problem in distributed computation. *IEEE Transactions on Software Engineering*, 1983.

[21] A. V. Goldberg, R. E. Tarjan, A New Approach to the Maximum Flow Problem. *Journal of the ACM*, 1988.

Definitions:

- Forward Labeling of a_{ij}:
  If n_i is labeled and n_j is not, and $u_{ij} > f(a_{ij})$, n_j gets labeled, and $\Delta_{ij} = u_{ij} - f(a_{ij})$
- Backward Labeling of a_{ij}:
  If n_i is labeled and n_j is not, and $f(a_{ij}) > 0$, n_j gets labeled, and $\Delta_{ij} = f(a_{ij})$

Algorithm:

1. Assign an initial flow $(f(a_{ij}) = 0$ for all arcs in $A)$
2. Mark s labeled and all other nodes unlabeled
3. Search for a node that can be labeled by either a forward or backward edge. If none found, flow is maximum, stop. If $n_j = t$ go to step 4, otherwise, repeat this step.
4. Backtrack the path computing the minimum Δ_{ij} used. If a_{ij} used a forward labeling, $f(a_{ij}) \longleftarrow f(a_{ij}) + \Delta_{ij}$. If a_{ij} used a backward labeling, $f(a_{ij}) \longleftarrow f(a_{ij}) - \Delta_{ij}$. Go to step 2.

Figure 1: Ford and Fulkerson Sequential Algorithm for Max-Flow [2]

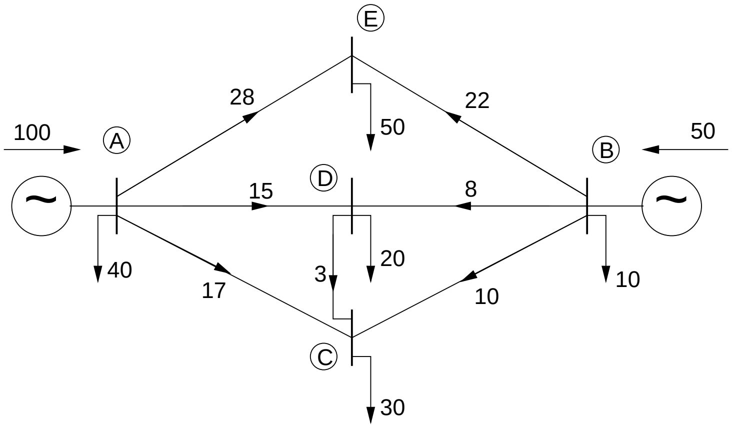
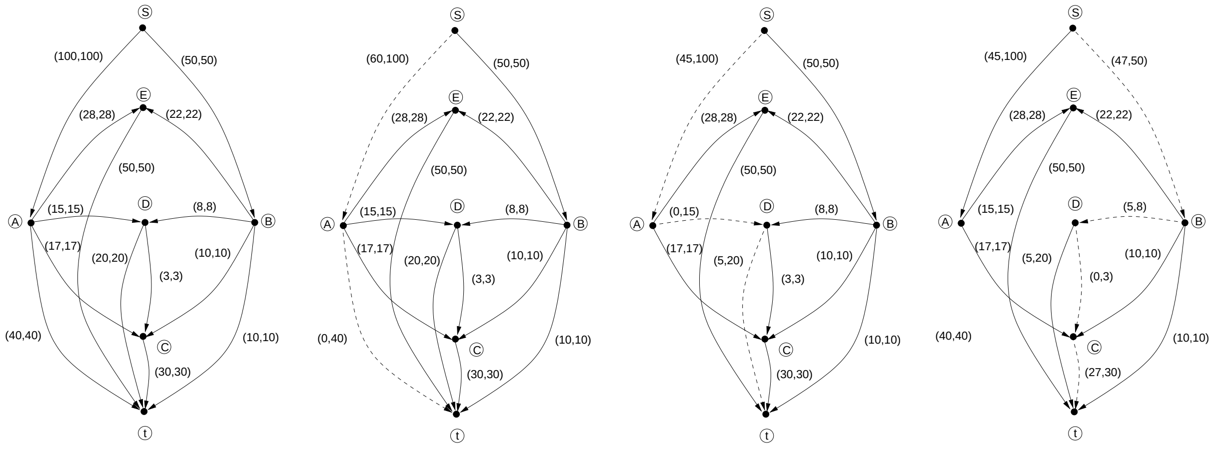
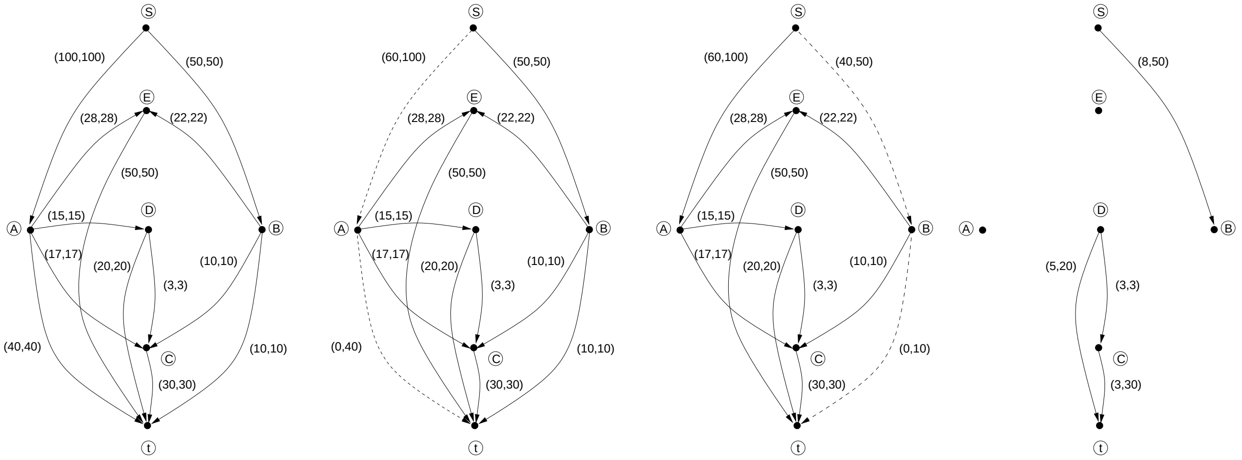


Figure 2: Example power system network



(a) (b) (c) (d)

Figure 3: Power network as a directed flow graph with virtual nodes s and t. Each arc is labeled (*remaining capacity*, *maximum flow*). (a) Initial flow. (b-d) Successive flow augmentations showing lines not at capacity.



(a) (b) (c) (d)

Figure 4: Power network as a directed graph with a loss of line B-D. Each arc is labeled (*remaining capacity*, *maximum flow*). (a) Initial flow with loss of line. (b-c) Successive flow augmentations (d) Final flow.

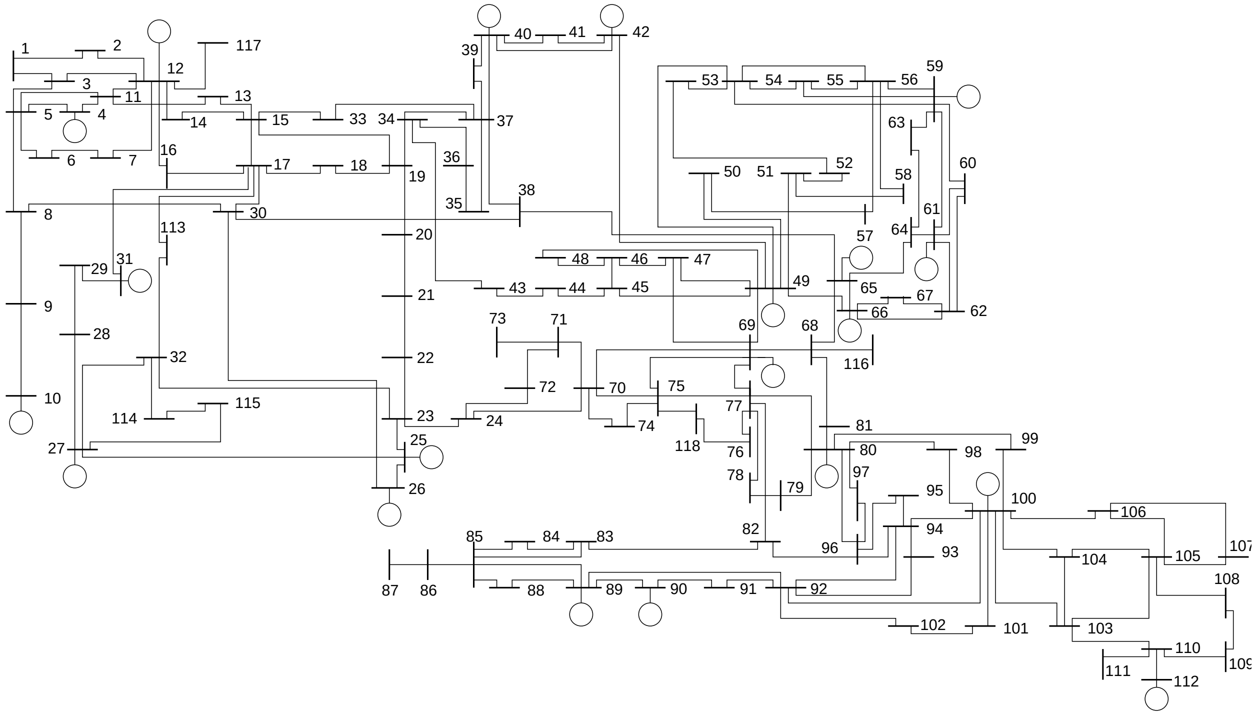


Figure 5: IEEE 118 bus test system

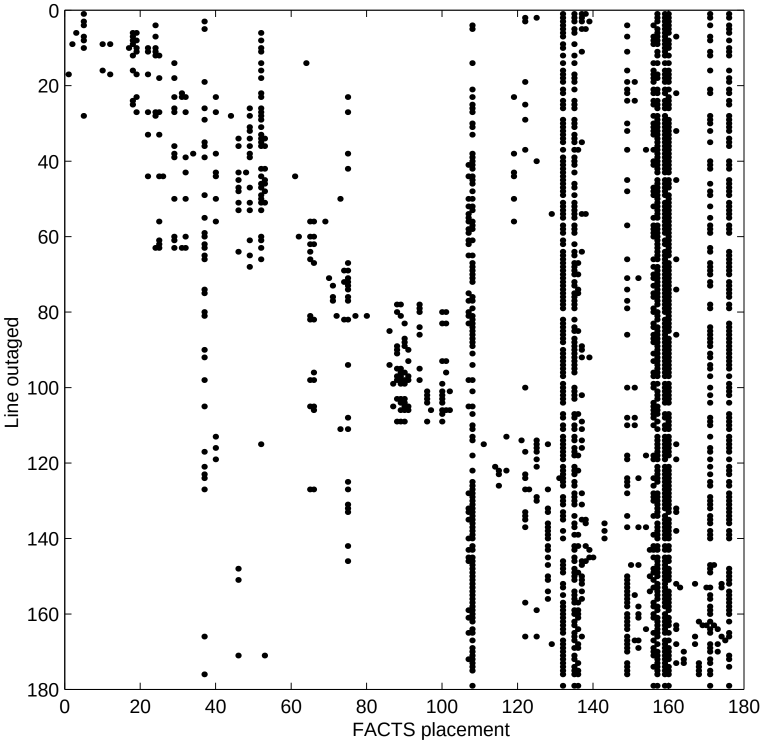


Figure 6: FACTS placement of 10 devices for each line outage in the 118 bus system.

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Distributed Max-Flow Algorithm:
Assign an initial flow ($f(a_{ij}) = 0$ for all arcs in $A$)
for each power source $s_i$, do in parallel
 1. Mark $s_i$ labeled.
 2. Select an unlabeled outgoing arc $n_j$. If there are no more unlabeled outgoing arcs, stop.
 3. Label node $s_i$ and the arc with $(s_i, \Delta_{ij})$.
 4. Explore the arc $a_{s_i j}$ with a probe$(s_i, \Delta_{ij})$.
 5. Wait for messages to return. If the return message is
      • path$(s_i, \Delta_{ij})$, set $f(a_{s_i j}) \leftarrow f(a_{s_i j}) + \Delta_{ij}$
      • blocked$(s_i)$: All paths to $t$ are blocked by other searches. Unlabel the arc $a_{s_i j}$. Go to step 2.
      • echo$(s_i)$: All paths to $t$ along $a_{s_i j}$ are filled to capacity. Go to step 2.
for each node, $n_l$
 1. Wait for a probe$(s_i, \Delta_{kl})$
 2. If the node is already labeled by $s_i$, return echo$(s_i)$ to the sender of probe. Go to step 1
 3. If the node is already labeled (by some $s_j$), return blocked$(s_i)$ to the sender of the probe. Go to step 1
 4. If the node is a sink node,
      • If $a_{lm}$ used a forward labeling, $f(a_{lm}) \leftarrow f(a_{lm}) + \Delta_{lm}$.
      • If $a_{lm}$ used a backward labeling, $f(a_{lm}) \leftarrow f(a_{lm}) - \Delta_{lm}$.
    clear labels for $s_i$, send path$(s_i, \Delta_{lm})$ to sender of probe, and go to 1.
 5. Select an unlabeled arc, $a_{lm}$.
 6. If there are no more unlabeled arcs that are not already at capacity, send echo$(s_i)$ to the sender of the probe, $n_k$, and clear the label for $s_i$ and go to 1.
 7. $\Delta_{lm} \leftarrow u_{lm} - f_{lm}$.
 8. Label the arc and send a probe of $(s_i, \Delta = \min(\Delta_{kl}, \Delta_{lm}))$.
 9. Wait for messages to return. If the return message from node $n_m$ is
      • path$(s_i, \Delta_{lm})$
          – If $a_{lm}$ used a forward labeling, $f(a_{lm}) \leftarrow f(a_{lm}) + \Delta_{lm}$.
          – If $a_{lm}$ used a backward labeling, $f(a_{lm}) \leftarrow f(a_{lm}) - \Delta_{lm}$.
        clear labels for $s_i$, set $\Delta_{kl} \leftarrow \Delta_{lm}$, and send path$(s_i, \Delta_{kl})$ to sender of probe.
      • blocked$(s_i)$, all paths to $t$ are blocked by other searches, clear labels for $s_i$ and send blocked$(s_i)$ to sender of probe.
      • echo$(s_i)$: All paths to $t$ are filled to capacity. Go to 5.
10. Go to 1.
```

Figure 7: Distributed Algorithm Using Multiple Sources for Max-Flow. Uses the same definitions as Figure 1.

Table 1: Chronically overloaded lines remaining after max-flow line placement and line settings in the presence of all single contingencies.

#	i	j	#	i	j
37	23	24	149	91	92
107	65	66	156	94	96
108	65	68	157	94	100
132	80	96	159	96	97
135	80	99	160	98	100
136	82	83	171	105	106
137	82	96	176	109	110

Table 2: FACTS Placement and Line Overload Results.

No. of FACTS	No. of Overloads
none	123
1	34
2	23
4	19
5	16
6	15
7	12
8	11
9	10
10	9